A country portfolio approach to solving currency invoicing

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Abstract

This paper develops a simple framework for computing equilibrium shares of trade currency invoicing in open economy dynamic stochastic general equilibrium models. The solution method follows closely to Devereux and Sutherland [2011]'s method in solving portfolio choice by applying information from second-order approximations of equilibrium conditions to solving zero-order portfolio shares. The framework is flexible enough to be extended to a Rotemberg sticky price model. To illustrate the approach, I use a simple symmetric two-country model and show that the results are consistent with existing theoretical findings on how monetary policy affects exchange rate pass-through.

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1 Introduction

This paper develops a simple and tractable framework for computing equilibrium shares of currency invoicing used in trade in open economy dynamic stochastic general equilibrium (DSGE) models. In the presence of price rigidity, the choice on trade invoicing currency plays an important role in model transmission mechanism and optimal monetary policy in an open economy. This is because the degree of exchange rate pass-through varies with different invoicing currencies. Therefore the outcome of relative prices and real allocations strongly depends on the invoicing currency choice. Two prevalent assumptions on trade invoicing currency have been widely applied in the new Keynesian macroeconomics. The conventional assumption, first adopted by Obstfeld and Rogoff [1995] from the Mundell-Fleming model in their contribution to the new open economy macroeconomics, assumes that exporters' prices are sticky in the currency of producers, which is called 'producer currency pricing'(PCP). The assumption of PCP implies full exchange rate pass-through and the law of one price holds. An alternative assumption developed in Betts and Devereux [2000] and Devereux and Engel [2003] presumes prices be sticky in the currency of local buyers, which is called 'local currency pricing'(LCP). In this case, exchange rate pass-through is zero, and there exist relative price dispersions between different destination markets for the same tradable good. As surveyed in a recent handbook chapter by Corsetti et al. [2010], the two assumptions yield very different equilibrium outcomes and implications on optimal monetary policy.

Despite the fruitful studies done based on PCP and LCP, recent empirical works on exchange rate pass-through and trade currency invoicing show neither hypothesis matches the reality. Empirical estimates on exchange rate pass-through is usually significantly different to 0 or 1, and there's clear evidence that strongly favors partial pass-through.[1] Data on trade invoicing currency also validates the above findings by showing that neither PCP nor LCP dominates in advanced economies. Models such as Corsetti et al. [2008] attempting to generate partial pass-through with staggered price setting or local input do not explain the pattern of invoicing currency. Furthermore, as demonstrated in Choudhri and Hakura [2015], a simple model with a reasonably exogenously imposed mix of PCP and LCP can explain the observed pass-through. Yet, studies on endogenous currency invoicing choice in a general equilibrium model with dynamic price adjustment are rare at best.

The aim of this paper is to develop a tractable framework that solves the share of trade currency invoicing in a general equilibrium model. As widely known in the literature on invoicing currency, standard perturbation method is not feasible as the equilibrium

[1] See, for example, Campa and Goldberg [2005], Anderton [2003], Bussire et al. [2014], Frankel et al. [2012], Choudhri and Hakura [2015]

invoicing share is indeterminate in the non-stochastic steady state. Intuitively, decisions on currency invoicing rely strongly on the motivation to hedge the risk of unexpected price movement, which can be only captured at approximations at least at the second-order. The contribution of this paper is to present a framework such that the endogenous currency invoicing problem shares the same exact numerical features in a standard portfolio choice problem, which allows for the application of the latter's solution method to the former. I build upon the numerical method developed in Devereux and Sutherland [2011] and Tille and van Wincoop [2010] by utilizing information from the second-order approximation to determine the zero-order component of currency invoicing share. The numerical solution is implemented based on a fixed point iteration and numerical root finding, which is a procedure utilized in Tille and van Wincoop [2010] in solving equilibrium portfolio.

This paper is closely related to the literature that has sought to explain the specific determinants of the currency denomination in exports. The numerical method developed in this paper mainly focuses the hedging motive in optimal currency invoicing, which shares similar intuition to Corsetti and Pesenti [2015], Devereux et al. [2004] and Novy [2006]. In particular, the endogenous currency invoicing share problem in the model is equivalent to Corsetti and Pesenti [2015]'s approach in choosing an optimal exchange rate pass-through indexation in a static setting. Thus the condition for determining currency invoicing (exchange rate pass-through) is similar. Relative to the literature, the main contribution of this paper is that the proposed framework in this paper can be easily extended to a dynamic model with gradual price adjustment, which is more applicable for generating equilibrium dynamics that match observed business cycle behavior. There are other determinants of the choice of invoicing currency found in the existing literature, such as 'coalescing motive' (Goldberg and Tille [2008]), 'bargaining weight' (Goldberg and Tille [2013]), and 'market structure' (Bacchetta and van Wincoop [2005],), which I shall leave extensions on incorporating these features in future work.

The paper is organized as follows. Section 2 presents a static partial equilibrium model on a firm profit maximization problem, and derives the key condition for determining the zero-order currency invoicing share solution. Section 3 extends the firm profit maximization problem to a dynamic model with price stickiness and shows how the same numerical method can still be applied. Section 4 demonstrates the numerical solution procedure with a standard two-country model and show that the effect of nominal stabilization on equilibrium invoicing share is consistent with the findings in the literature. Section 5 concludes.

2 A static partial equilibrium model

In this section, I illustrate how the optimal currency invoicing problem is analogous to a standard financial portfolio problem in a simple static framework. I focus on a partial equilibrium setting based on a firm's profit maximization problem. This is because currency invoicing affects the economy mainly through the channel of exchange rate pass-through on export(import) price, thus examining a firm's profit maximization problem is sufficient enough to demonstrate the intuition. The model shares standard features that are widely used in modern macroeconomic literature: imperfect competition, nominal rigidity, and forward-looking price setting behavior by firms. The specific functional forms and variable notations of the profit maximization problem are constructed so that they can be embedded into the general equilibrium example later. The key element of this model is that firms can optimally decide the degree of pass-through by adjusting the share of currency invoicing.

2.1 The model

Assume a country produces a tradable good that sells in the domestic market and is also exported to a foreign market. The traded good consists of varieties that are defined over a continuum of unit mass. Each variety is produced by a single firm that faces monopolistic competition in the goods market and takes factor prices as given. For the sake of simplicity, I assume all firms have identical production functions that are of constant returns to scale.

To introduce nominal rigidity into the model, I assume that firms post their prices before knowing the realization of shocks that affect the economy. Given the monopolistic competition structure of the goods market, firms stand ready to meet demand at a given price after the realization of shocks. I consider that each individual firm optimally chooses two nominal prices that differ only in their currency denomination. The price that is denominated in the domestic market currency is PCP, and the price that is denominated in the foreign market currency is LCP. The main difference between the two is that following any unexpected fluctuation in the exchange rate, the exchange rate movement is fully "passed-through" in PCP, while having no effect on LCP.

For a firm that produces a variety h, its domestic price is always equal to PCP. Thus, the firm's nominal profit $\Pi_H(h)$ from sales in the domestic market is defined as:

$$\Pi_H(h) = (P_p(h) - MC)(\frac{P_p(h)}{P_H})^{-\theta} Y_H, \tag{1}$$

where $P_p(h)$ is the PCP set by the firm, Y_H is the domestic demand on the tradable good, θ is the constant price elasticity of demand. Assuming that demand over different varieties are weighted equally, I define P_H as the domestic price index:

$$P_H = [\int_0^1 P_p(h)^{1-\theta} dh]^{\frac{1}{1-\theta}} \tag{2}$$

As for the sales in the foreign market, I consider a general and flexible specification where the foreign price is a weighted sum of PCP and LCP. Before the realization of shocks, the firm optimally adjusts the weight, which determines the exchange rate pass-through on the actual local foreign price that's charged on foreign buyers. This specification is similar to the currency basket approach used in Goldberg and Tille [2008]. Formally, I define the foreign price $P_f(h)$, which is measured in the foreign currency as:

$$P_f(h) = S(h)P_p(h)/e + (1 - S(h))P_l(h), \tag{3}$$

where $P_l(h)$ is LCP, e is the nominal exchange rate expressed as the domestic currency per unit of the foreign currency, and $S(h)$ is the weight assigned to PCP in the foreign price $P_f(h)$. Since in equilibrium the zero-order components of $P_p(h)/e$ and $P_l(h)$ are exactly the same, $S(h)$ also corresponds to the share of domestic currency used in trade invoicing currency. As $S(h)$ is a choice variable in the firm's profit maximization problem, this flexible specification provides a tractable way in showing the share of currency invoicing in equilibrium. The standard assumption of pure PCP(LCP) used in the literature corresponds to $S(h) = 1(S(h) = 0)$. The nominal profit $\Pi_F(h)$ from sales in the foreign market can be defined as:

$$\Pi_F(h) = (eP_f(h) - MC)(\frac{P_f(h)}{P_H^*})^{-\theta} Y_F, \tag{4}$$

where Y_F is the foreign demand, and P_H^* is the foreign price index:

$$P_H^* = [\int_0^1 P_f(h)^{1-\theta} dh]^{\frac{1}{1-\theta}}. \tag{5}$$

Before the realization of shocks, the firm optimally decides $P_p(h)$, $P_l(h)$, and $S(h)$ to maximize the sum of expected profit from both the domestic market and the foreign market. Conditional on these choice variables, the value of $P_f(h)$ is only dependent on the realized value of exchange rate. All other variables are taken as given, and the distribution of these variables can be imposed exogenously or be determined in a general equilibrium. Thus, the profit maximization problem is defined as:

$$\max_{P_p(h),P_l(h),S(h)} E[\Pi_H(h) + \Pi_F(h)]. \tag{6}$$

Maximizing this expression leads to the following first-order conditions for the firm's choice variables:[2]

$$P_p = P_H = \frac{\theta}{\theta - 1}\frac{E[MC(Y_H + \frac{SP_p}{eP_f}Y_F)]}{E[(Y_H + SY_F)]} \tag{7}$$

$$P_l = \frac{\theta}{\theta - 1}\frac{E[MC\frac{(1-S)P_l}{P_f}Y_F]}{E[e(1-S)Y_F]} \tag{8}$$

$$E[P_p(1 - \frac{\theta}{\theta - 1}\frac{MC}{eP_f})Y_F] = E[eP_l(1 - \frac{\theta}{\theta - 1}\frac{MC}{eP_f})Y_F]. \tag{9}$$

$$P_f = P_H^* = SP_p/e + (1 - S)P_l \tag{10}$$

Equation 7 and 8 are the standard first-order conditions for PCP and LCP, with the only difference being that the foreign demand is now weighted by the share of currency invoicing. Equation 9 is the key first-order condition that determines the share of currency invoicing S in equilibrium. The three first-order conditions combined with the definition of foreign price in equation 10, form a system of 4 equations that solves the 4 unknown variables (P_p, P_l, S, P_f) in the firm's profit maximization problem.

2.2 Solving for the zero-order currency invoicing

It is well known in the currency invoicing literature that invoicing shares are not uniquely defined when there are no shocks. This is because the domestic currency values of PCP and LCP are equal in the non-stochastic steady state, thus adjusting invoicing shares does not matter in a world without shocks. Furthermore, due to certainty equivalence, a first-order approximation of the system of equations is not enough to determine invoicing shares either. In general, unless an exact analytical solution can be found, the standard approach is to rely on a quadratic approximation to capture the second-order effects that determine invoicing shares.[3]

The necessity for the quadratic approximation lies in the fact that firms' motivation for currency invoicing is to minimize the risk of uncertain fluctuations in the foreign price, as detailed below. This can only be captured by approximations at the second-order or

[2]The firm specific variety indicator h is removed since in equilibrium all firms make the same decisions.
[3]See, for example, Goldberg and Tille [2008] and Engel [2006].

higher, as it requires information on variances and covariances. In the macroeconomic financial portfolio literature, similar technical issues arise as equilibrium portfolios are determined by the variance-covariance terms of excess asset returns and the stochastic discount factor. The similarities between the endogenous currency invoicing problem and the endogenous portfolio problem allow for the application of existing numerical solution method from the latter to the former.

In the seminal works by Devereux and Sutherland [2011] (hereafter 'DS') and Tille and van Wincoop [2010], both papers present a straightforward and tractable method in deriving the zero-order portfolio based on information from higher order approximations on the portfolio Euler equations. For the sake of brevity, I shall follow the DS method and describe the key mechanism behind their solution procedure for the zero-order portfolio, and ask readers to refer to their paper for detail. In their model, the portfolio variables affect the first-order solutions of any non-portfolio variables only through their zero-order solutions. To pin down the zero-order solutions of the portfolio variables, their numerical method utilizes information from the second-order approximations of the portfolio Euler equations. These second-order approximations can be derived based on the variances and covariances of first-order components of the non-portfolio variables. This creates a fixed point problem: the zero-order portfolio maps into the first-order solutions of non-portfolio variables, which maps back into the zero-order portfolio through the second-order approximations of portfolio Euler equations. Such problem can easily be tackled numerically through iteration.[4] Solutions for the first or higher-order approximation of portfolio variables can be achieved by applying a similar procedure to the portfolio Euler equation at the third-order approximation or higher.[5]

As the DS solution method relies on approximation around a non-stochastic steady state allocation, it shares similar deficiencies with other approximation methods that are local. Furthermore, In Rabitsch and Stepanchuk [2014] and Rabitsch et al. [2015], they discuss that there are two potential problems specific to the DS solution, causing it to be less accurate compared to a global solution method. The first problem is that in an endogenous portfolio model with incomplete financial markets, the net foreign asset position is not stationary unless an otherwise arbitrary stationary inducing device is introduced. When applying the same solution method to the currency invoicing problem, this is not an issue since firm's profit maximization problem does not introduce non-stationarity. The second problem is that the DS solution does not take into account the effect of the size of shocks on the portfolio solution. Due to the stability of currency

[4]In Devereux and Sutherland [2011], they are able to obtain simple analytical solutions by making use of the insight that excess returns can be treated as i.i.d. random variables.

[5]Samuelson [1970] states that to derive any N-th order solution of portfolio variables, it is necessary to approximate the portfolio problem to an order of N+2.

invoicing share documented in various empirical work, I mainly focus on the zero-order component of the solution. Thus the interpretation of currency invoicing solution is a limiting case where the size of shock is arbitrarily small, which is independent of the actual size of shocks.

To adopt a similar numerical method to the currency invoicing problem, I first state two properties in the currency invoicing problem that are equivalent to the properties in the financial portfolio problem discussed in Devereux and Sutherland [2011]. In what follows, the zero-order variables are represented with a bar, while the first-order log-deviation of variables are represented with a hat. I call all variables other than S as non-share variables. I redefine $MC/(eP_f)$ as rmc_f, which I shall call it foreign real marginal cost. The two properties are stated as follows

Property 1. It is sufficient to evaluate the second-order approximation of equation 9 by deriving the expressions for the first-order solutions of non-share variables. This is because the second-order approximation of equation 9 is $E[(\hat{P}_p - \hat{e} - \hat{P}_l)r\hat{m}c_f] = 0$, and the left hand side of this equation is a product of first-order solutions of non-share variables.

Property 2. S affects the first-order approximations of non-share variables only through its zero-order component $\bar{S}$. To see this, notice that in the first-order conditions, S only appears in equation 7, 8, and 10. The first-order approximations of equation 7 and 8 do not contain S.[6] The first-order approximation of the remaining equation 10 is $\hat{P}_f = \bar{S}(\hat{P}_p - \hat{e}) + (1 - \bar{S})\hat{P}_l$, which relies only on the zero-order component $\bar{S}$.

With the above two properties in mind, it is straightforward to apply the numerical method used in the financial portfolio literature for solving the zero-order component of currency invoicing. Based on property 2, $\bar{S}$ maps into the first-order solutions of non-share variables, and by property 1 these non-share variables will map back to $\bar{S}$ through the second-order approximation on equation 9: $E[(\hat{P}_p - \hat{e} - \hat{P}_l)r\hat{m}c_f] = 0$. With any standard solution method for linear rational expectations models, the first-order solution for non-share variables can be found conditional on a given $\bar{S}$, and what's left is to find the $\bar{S}$ that satisfies $E[(\hat{P}_p - \hat{e} - \hat{P}_l)r\hat{m}c] = 0$, which can be done numerically.

The intuition for how currency invoicing is optimally determined is rather straightforward. Notice that applying the first-order approximation of equation 7 and 8 to the second order approximation of equation 9 gives the following condition:

$$Cov(\hat{e} - E(\hat{e}), r\hat{m}c) = 0. \tag{11}$$

[6] Log-linearizion to equation 7 and 8 leads to $\hat{P}_p = E[\hat{MC}]$ and $E[\hat{P}_f] = E[\hat{MC}] - E[\hat{e}]$

This condition captures the key insight in why firms adjust their invoicing share: they try to adjust the share of currency invoicing such that unexpected movement in the exchange rate is uncorrelated with movement in the foreign markup. As prices are set in advance based on expectations on future shocks, the uncertain adjustment in the foreign price is purely based on unexpected exchange rate movement, with the degree of exchange rate pass-through dependent on the share of currency invoicing. Knowing this, firms optimally hedge themselves from unexpected exchange rate movement by choosing the share of currency invoicing that satisfies equation 11. The similarity between how firms hedge themselves against exchange rate risk, and how investors hedge their portfolios against risky asset returns is what allows for the application of similar numerical methods in the first place. In a more restrictive model, Corsetti and Pesenti [2015] shows the same condition analytically, and equation 11 can be seen as a more generalized approximation with the same insight.

3 Extension to a Rotemberg sticky price model

I now demonstrate how the same solution method can be extended to a dynamic firm profit maximization problem with gradual price adjustment. As is well known in the New Keynesian literature, models with prices set one period ahead fall short in matching the data, as the persistency of price adjustment strongly depends on the persistency of underlying shocks. Thus, I follow Rotemberg [1982] and assumes that firms need to pay quadratic adjustment costs when changing prices. The timing of the firm's choice variables is as follows. In each period t, the foreign price is dependent on PCP and LCP decided in the current period, and the currency invoicing share decided in the previous period $t-1$:

$$P_{f,t}(h) = S_{t-1}(h)P_{p,t}(h)/e_t + (1 - S_{t-1}(z))P_{l,t}(h). \tag{12}$$

The assumption on the timing is to allow for the role of hedging motives in determining the share of currency invoicing. If firms are allowed to decide the share of currency invoicing simultaneously with the prices, this gives too much freedom as any desired foreign price can be achieved by adjusting the currency invoicing share, even without adjustment in PCP or LCP. Furthermore, because only the next period foreign price is directly affected by the currency invoicing share, this timing assumption allows me to introduce a more realistic price dynamic, while retaining the static structure of the firm's decision on currency invoicing.

Similar to the static framework, I assume that the price in the domestic market is always equal to PCP. The nominal profit generated from sales in the domestic market in period t is expressed as:

$$\Pi_{H,t}(h) = (P_{p,t}(h) - MC_t)(\frac{P_{p,t}(h)}{P_{H,t}})^{-\theta} Y_{H,t} - \frac{v_p}{2}(\frac{P_{p,t}(h)}{P_{p,t-1}(h)} - 1)^2 P_{p,t}(h)(\frac{P_{p,t}(h)}{P_{H,t}})^{-\theta} Y_{H,t}. \quad (13)$$

where the term on the right hand side of the equation is the nominal cost due to price adjustment, determined by a parameter v_p (price is flexible if $v_p = 0$). As for the sales in the foreign market, I assume that adjustment costs due to changes in PCP and LCP are weighted by how much nominal revenue is generated by each price. Formally, the nominal profit from sales in the foreign market is defined as:

$$\begin{aligned} \Pi_{F,t}(h) =& (e_t P_{f,t}(h) - MC_t)(\frac{P_{f,t}(h)}{P^*_{H,t}})^{-\theta} Y_{F,t} \\ & - \frac{v_p}{2}(\frac{P_{p,t}(h)}{P_{p,t-1}(h)} - 1)^2 S_{t-1}(h) P_{p,t}(h)(\frac{P_{f,t}(h)}{P^*_{H,t}})^{-\theta} Y_{F,t} \\ & - \frac{v_l}{2}(\frac{P_{l,t}(h)}{P_{l,t-1}(h)} - 1)^2 (1 - S_{t-1}(h)) e_t P_{l,t}(\frac{P_{f,t}(h)}{P^*_{H,t}})^{-\theta} Y_{F,t}, \end{aligned} \quad (14)$$

I allow for the parameters (v_p and v_l) that determine the price adjustment cost to differ between PCP and LCP. This specification allows for another motive in the firm's optimal currency invoicing decision: the firm would lower the share on one invoicing currency if the expected adjustment cost of price in that currency is expected to be large. Given the expression for the profit in every period t, the firm maximizes lifetime expected profit, subject to a common stochastic discount factor $Q_{t,\tau}$:

$$\max_{P_{p,t}(z), P_{l,t}(z), S_t(z)} E_t \sum_{\tau=t}^{\infty} Q_{t,\tau}[\Pi_{H,\tau}(h) + \Pi_{F,\tau}(h)]. \quad (15)$$

Due to the complexity in the expressions, I relegate the details of the first-order conditions and their first-order approximations to the Appendix. To apply the same numerical method to the Rotemberg price model, I state two properties that are similar to those presented in the static setting, with a slight adjustment in the equations:

Property 1. It is sufficient to evaluate the second-order approximation of the first-order condition that determines the currency invoicing share by deriving the expressions for the first-order solutions of non-share variables. This is because the second-order approximation of equation 39 in the Appendix is:

$$E_t[(\hat{\Delta}_{pl,t+1}) r\hat{m}c_f - \frac{v_p}{2(\theta - 1)} \hat{\pi}^2_{p,t,t+1} + \frac{v_l}{2(\theta - 1)} \hat{\pi}^2_{l,t,t+1}] = 0, \quad (16)$$

where $\Delta_{pl,t+1} = P_{p,t+1} - e_{t+1} P_{l,t+1}$ is the measure of price misalignment between

PCP and LCP, $\pi_{p,t} = \frac{P_{p,t}}{P_{p,t-1}}$, and $\pi_{l,t} = \frac{P_{l,t}}{P_{l,t-1}}$. Since every term in this expression is a product of two first-order solutions, it can be evaluated based on the first-order approximation of non-share variables.

Property 2. S_t affects the first order approximation of non-share variables only through its zero-order component $\bar{S}$. To prove this, equation 40, 41 and 42 in the Appendix are the linear approximations of the first-order conditions. None of them contains S_t that is of order higher or equal to 1. With the assumption that the nominal adjustment costs on PCP and LCP are also dependent on S_t, in a fully specified general equilibrium model S_t would also enter the market clearing condition. However, up to a first-order approximation it is not relevant, since the adjustment cost terms take effect at an approximation at the second-order.

Equation 16 is the key condition that determines $\bar{S}$. As is similar to the static model, the above two properties allow me to characterize the solution for optimal invoicing share as a fixed point problem. However, due to the price stickiness caused by the adjustment costs, the expression is more complex compared to the static example. The first complexity arises due to the introduction in the dynamic of price adjustment. Unlike the static example, where the price misalignment between PCP and LCP is solely caused by unexpected movement in the exchange rate, movements in PCP and LCP in the Rotemberg sticky price model also contribute to adjustment in the price misalignment variable $\Delta_{pl,t+1}$. Thus, firms do not only hedge the risk in the exchange rate movement, but also need to take into account the risk of both PCP and LCP adjustment. Furthermore, as the decision on currency invoicing share also determines the weight on the adjustment costs on PCP and LCP, this gives firms an incentive to optimize the weight to minimize expected adjustment costs, as shown in the two terms containing $\pi_{p,t+1}$ and $\pi_{l,t+1}$ in equation 16.

A numerical challenge in using equation 16 for determining the equilibrium currency invoicing share is that the equilibrium variables in a dynamic model respond to state variables. Thus, even though the condition for the optimal invoicing share remains to be the same in every period, the implication on the solution can be different based on the state of the economy. Thus, for the sake of tractability, I restrict the numerical solution of invoicing share to be determined by equation 16 at the non-stochastic steady state. Given that the main objective of this solution method is in solving for the zero-order component of S_t, which is a term independent of the underlying state, I believe the assumption is valid.[7]

[7] The interpretation for the zero-order solution is that it is an equilibrium in a 'near non-stochastic' state where shocks are arbitrarily small, as in Devereux and Sutherland [2011]. This also validates the restriction for focusing on solving $\bar{S}$ at the steady state.

4 A general equilibrium example

I demonstrate this solution method on trade currency invoicing by applying it in a stylized symmetric two-country sticky price general equilibrium model. In the model, I revisit the classical question on how monetary policy affects the exchange rate pass-through, where exchange rate pass-through is mainly determined by the endogenous adjustment in firms' currency invoicing. The purpose of the model is not to empirically match the observed exchange rate pass-through in the data, as doing so would require a much more complex model structure with many underlying shocks to generate empirically plausible moments for the non-share variables. The novel feature of the model is that sticky price is modeled à la Rotemberg [1982], as opposed to price set one period in advance as commonly used in the currency invoicing literature.

The model consists of a Home(H) country and a Foreign(F) country, where each country specializes in one type of tradable good. Each country is populated by households defined over a continuum of unit mass, which I shall index by j in the Home country and j^* in the Foreign country. Households supply labor to firms, and consume a basket of tradable goods produced in both countries. Due to the imposed symmetry between the two countries, I shall only discuss Home country in the following, unless necessary otherwise. The variables related to the Foreign country are marked with an asterisk (*).

4.1 Basic setup

Household j's lifetime expected utility $U_t(j)$ in period t is defined as:

$$U_t(j) = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}[\frac{C_\tau(j)^{1-\sigma}}{1-\sigma} - \chi \frac{L_\tau(j)^{1+\kappa}}{1+\kappa}], \tag{17}$$

where β is the discount factor, $C(j)$ is the consumption index, and $L(j)$ is the labor effort. Consumption index $C(j)$ is a basket of Home and Foreign goods with constant elasticity of substitution (CES) between the two tradable goods:

$$C_t(j) = [\alpha^{\frac{1}{\phi}} C_{H,t}(j)^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} C_{F,t}(j)^{\frac{\phi-1}{\phi}}]^{\frac{\phi}{\phi-1}}, \tag{18}$$

where ϕ is the intratemporal elasticity of substitution, and $\alpha > \frac{1}{2}$ measures the degree of home bias in consumption. The consumption indices of both Home and Foreign tradable goods are defined by CES baskets of varieties produced in the respective country:

$$C_{H,t}(j) = [\int_0^1 C_t(h,j)^{\frac{\theta-1}{\theta}} dh]^{\frac{\theta}{\theta-1}}, \qquad C_{F,t}(j) = [\int_0^1 C_t(f,j)^{\frac{\theta-1}{\theta}} df]^{\frac{\theta}{\theta-1}}, \tag{19}$$

where $C_t(h,j)$ ($C_t(f,j)$) are the consumption of Home household j on Home variety h (Foreign variety f) in period t. Let $P_{H,t}(h)$ and $P_{F,t}(f)$ be the prices of Home and Foreign varieties. The demand functions for Home and Foreign goods and the varieties within each basket can then be derived as:

$$C_{H,t}(j) = \alpha(\frac{P_{H,t}}{P_t})^{-\phi}C_t(j), \qquad C_{F,t}(j) = (1-\alpha)(\frac{P_{F,t}}{P_t})^{-\phi}C_t(j), \tag{20}$$

$$C_t(h,j) = (\frac{P_{H,t}(h)}{P_{H,t}})^{-\theta}C_{H,t}(j), \qquad C_t(h,j) = (\frac{P_{F,t}(f)}{P_{F,t}})^{-\theta}C_{F,t}(j), \tag{21}$$

where the price indexes are defined as:

$$P_t = [\alpha P_{H,t}^{1-\phi} + (1-\alpha)P_{F,t}^{1-\phi}]^{\frac{1}{1-\phi}} \tag{22}$$

$$P_{H,t} = [\int_0^1 P_{H,t}(h)^{1-\theta}dh]^{\frac{1}{1-\theta}}, \qquad P_{F,t} = [\int_0^1 P_{F,t}(f)^{1-\theta}df]^{\frac{1}{1-\theta}}. \tag{23}$$

Household j maximizes equation 17 subject to a flow budget constraint. In each period, household consumption expenditure P_tC_t is financed by wage income $W_tL_t(j)$, dividends from firm profit $\Pi_t(j)$, and returns from holdings of financial asset. I assume complete financial market where the household has access to a full set of state-contingent (Arrow-Debreu) securities. The flow budget constraint in period t for the household is:

$$P_tC_t(j) + E_tD_{t,t+1}B_{t+1}(j) = W_tL_t(j) + B_t(j) + \Pi_t(j), \tag{24}$$

where $B(j)_{t+1}$ is the holdings of a portfolio of state contingent nominal bonds that promises to pay one unit in domestic currency in period $t+1$ if a specified state is realized. $D_{t,t+1}$ is the price of such bond, denominated in the domestic currency. Based on the assumption of complete financial market, in equilibrium it implies the following risk sharing condition:

$$\frac{C_t^{-\sigma}}{C_t^{*-\sigma}} = \frac{P_t}{e_tP_t^*}. \tag{25}$$

For a Home firm producing variety h, the production function is assumed to be linear in labor

$$Y_t(h) = A_tL_t(h), \tag{26}$$

where A_t is a country specific level of productivity that follows an AR(1) process with standard deviation σ_A and persistence parameter ρ_A. The firm takes wage as given and hire labor from a perfectly competitive market, thus the marginal cost is the same for all firms and it's equal to:

$$MC_t = \frac{W_t}{A_t}. \tag{27}$$

Wage rate is flexible, and households take the wage rate as given and supply labor services under perfect competition. Labor supply is determined by the intratemporal condition between consumption and labor:

$$\frac{\chi L_t^{\kappa}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \tag{28}$$

The profit maximization problem essentially follows the same structure presented in section 3. I focus on a symmetric equilibrium where all firms within a country charge the same price, and choose the same currency invoicing share. Thus the variety index h can be dropped in the equilibrium. This implies that $P_{p,t}(h) = P_{p,t} = P_{H,t}$, $P_{l,t}(h) = P_{l,t}$, $P_{f,t}(h) = P_{f,t} = P^*_{H,t}$, and $S_t(h) = S_t$. Utilizing the imposed symmetry in the equilibrium, the aggregate resource constraint for Home production then can be written as:

$$Y_{H,t}(1 - \frac{v_p}{2}(\frac{P_{p,t}}{P_{p,t-1}} - 1)^2) = C_{H,t} \tag{29}$$

$$Y_{F,t}[1 - \frac{v_p}{2}(\frac{P_{p,t}}{P_{p,t-1}} - 1)^2 \frac{S_{t-1}P_{p,t}}{P_{f,t}} - \frac{v_l}{2}(\frac{P_{l,t}}{P_{l,t-1}} - 1)^2 \frac{(1 - S_{t-1})P_{l,t}}{P_{f,t}}] = C^*_{H,t}, \tag{30}$$

where after the payment for the adjustment costs, the supply of Home goods has to be equal to consumption demand in the two countries.

I assume that monetary authorities in the model target the nominal interest R_t on the nominal bonds denominated in their countries' currencies. The target interest rate follows a simple Taylor rule that only reacts to the CPI inflation rate:

$$R_t = \bar{R}(\frac{P_t}{P_{t-1}})^{\delta} exp(\zeta_t), \tag{31}$$

where $\delta > 1$, and ζ_t is a zero mean i.i.d. monetary policy shock with standard deviation σ_ζ. The choice for choosing CPI in the Taylor rule is that it represents the actual practice done in central banks following inflation targeting policies. Furthermore, as demonstrated in Engel [2011], targeting CPI can yield preferable welfare compared to targeting PPI in the presence of incomplete pass through.

4.2 Numerical solution

The objective of this model example is to demonstrate the solution approach on solving the zero-order currency invoicing share. I reexamine a long-standing question in open-economy macroeconomics on how monetary policy can determine currency denomination. To the best of my knowledge, general equilibrium research on this topic has only been done in a setting where firms set price one period ahead. Thus, the model example contributes to the existing research with an extension that incorporates a more realistic price transmission mechanism. I focus on two particular aspects of monetary policy that relates to inflation stabilization: the feedback parameter in the Taylor rule δ, and the standard deviation of monetary policy shock σ_ς. δ corresponds to central banks' anti-inflationary policy stance, while σ_ς measures the credibility on whether central banks are able to deliver a consistent monetary policy.

Since only the zero-order components of currency invoicing share $[S_t, S_t^*]$ affect the first-order solutions for all the non-share variables. Thus, conditional on a given $[\bar{S}, \bar{S}^*]$, the non-share variables can be solved using any standard solution method for linear rational expectations model. Formally, the model solution is a set of non-time varying policy functions that maps endogenous variables x_t to state variables s_t and exogenous shocks z_t. Thus, the first-order solution for the non-share variables can be written as:

$$\hat{x}_t = \bar{x} + B_x \hat{s}_{t-1} + B_z \hat{z}_t \tag{32}$$

where the coefficient B_x and B_z are dependent on $[\bar{S}, \bar{S}^*]$.

In equilibrium, zero-order currency invoicing share $[\bar{S}, \bar{S}^*]$ satisfies the condition in equation 16. Given the first-order solution for the non-share variables, for Home country and Foreign country equation 16 is equivalent to the following two conditions at the steady state:

$$H = B_{rmc_f,z} \Sigma B_{\Delta_{pl},z}^{\intercal} - \frac{v_p}{2(z-1)} B_{\pi_p,z} \Sigma B_{\pi_p,z}^{\intercal} + \frac{v_l}{2(z-1)} B_{\pi_l,z} \Sigma B_{\pi_l,z}^{\intercal} = 0 \tag{33}$$

$$H^* = B_{rmc_f^*,z} \Sigma B_{\Delta_{pl}^*,z}^{\intercal} - \frac{v_p}{2(z-1)} B_{\pi_p^*,z} \Sigma B_{\pi_p^*,z}^{\intercal} + \frac{v_l}{2(z-1)} B_{\pi_l^*,z} \Sigma B_{\pi_l^*,z}^{\intercal} = 0 \tag{34}$$

where the left subscript on the coefficient matrix B_z indicates the corresponding row that represents the subscript, Σ stands for the variance-covariance matrix of the underlying shocks, and $\intercal$ stands for transpose. As B_z is a non-linear function of $\bar{S}$, I apply a standard non-linear solver that jointly solves equation 33 and 34 through iteration. Therefore, the full dynamic system of the model is solved as follows:

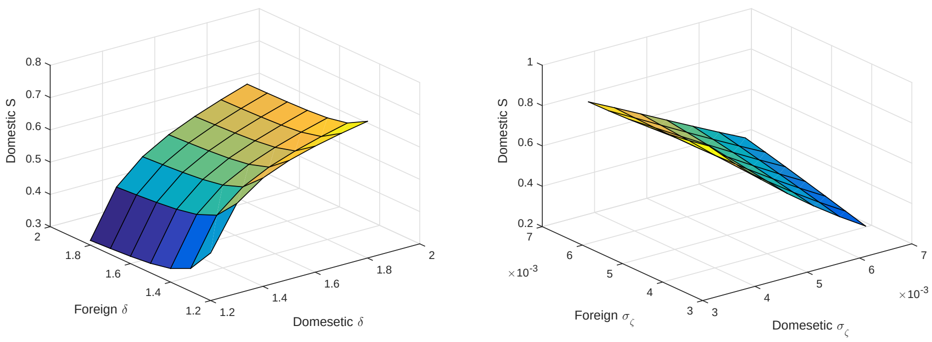
1. Select an error tolerance ξ as the stopping criterion, and an initial guess for $[\bar{S}, \bar{S}^*]$
2. Solve the first-order solution of non-share variables conditional on the guess
3. Evaluate equation 33 and 34
4. If $|H| < \xi$, and $|H^*| < \xi$, then $[\bar{S}, \bar{S}^*]$ is the solution for the zero-order currency invoicing share, as is the result from step 2 for the first-order solution on the non-share variables. If not, then update the guess and go back to step 2.

While the above solution procedure solves for $[\bar{S}, \bar{S}^*]$ in the $\mathbb{R}^2$ space, to be realistic I focus on a constrained solution with constraints $0 \leq \bar{S} \leq 1$ and $0 \leq \bar{S}^* \leq 1$. From a portfolio perspective, a currency invoicing share which is less than 0 or greater than 1 means that the exporter is essentially 'holding a negative position' on a certain currency for pricing in the foreign market, which is neither empirically feasible nor theoretically appealing. However, depending on the parameterization of the underlying shock process, it is possible that the solution that would satisfy both equation 33 and 34 would fall out of the constraint of $[0, 1]$. If such solution emerges, I restrict the country with unrealistic currency invoicing share on the boundary based on the hypothetical solution, and apply the same iterative procedure again to the other country. In this case, the interpretation of the constrained country would be that given the constraint where currency invoicing share has to be within the region of $[0, 1]$, firms do their best hedging the price misalignment and pick absolute PCP or LCP to minimize the absolute value on equation 33 or 34.

4.3 Calibration and result

The model is calibrated at a quarterly frequency. Most of the parameter values are consistent with the parameterization in Corsetti et al. [2010]. The discount factor β is assumed to equal to 0.99, the relative risk aversion σ is equal to 2, and the inverse elasticity of substitution on labor κ is equal to 1.5. The Armington elasticity between Home and Foreign tradable goods ϕ set to 2, and the elasticity of substitution between varieties θ is set to 6. I choose a value of 0.8 for the degree of home bias α. As for the scale parameter that determines the cost of adjusting prices, I follow Bilbiie et al. [2014] in setting υ_p and υ_l equal to 80. For productivity shocks, I let the persistence parameter ρ_A be equal to 0.95, and the standard deviation σ_A equal to 0.007. The cross-country correlation in productivity shocks is set as 0.25. I initially choose the inflation response parameter δ in the monetary policy interest rule to be equal to 1.5 , as in the simple version of the Taylor rule. I let the standard deviation of monetary policy shock σ_ζ to be 0.005 as my benchmark value, which is quite large compared to estimates based on medium-sized New Keynesian model by Smets and Wouters [2003, 2005, 2007]. This is due to the simple underlying shock process in the model example, and I purposely pick a rather large standard deviation so to generate an equilibrium currency invoicing share $\bar{S} = \bar{S}^* = 0.58$, which is close to the average values of the shares of invoicing calculated for advanced countries in Choudhri and Hakura [2015], where the value is 0.54. A more realistic model with a larger number of structural shocks may not require such large volatilities in the monetary shocks to generate the required currency invoicing share observed in data.

To demonstrate the consequences of varying monetary policy on equilibrium currency invoicing share, I solve the model with variations of δ and σ_ζ around the parameterization value for both countries. Figure 1 reports the relationship between zero-order domestic currency invoicing share $\bar{S}$ and the monetary policy parameters δ and σ_ζ. Each panel shows variations in one of the parameters, while keeping the other parameter at the benchmark value. A key result in both panels is that exporters tend to invoice in the currency from a country with better nominal stability, regardless whether nominal stability is due to a more anti-inflationary stance from a higher δ, or a more consistent monetary policy rule based on a lower σ_ζ. Furthermore, simultaneous increase in monetary stabilization in both countries leads to a worldwide increase in the exchange rate pass-through. The result is consistent with the theoretical findings on the role of monetary policy on exchange rate pass-thorough (Devereux et al. [2004], Corsetti and Pesenti [2015]).



(a) Anti-inflationary stance (b) Monetary policy credibility

Figure 1: Equilibrium zero-order Home currency invoicing share

5 Conclusion

This paper develops a simple framework for endogenous trade invoicing currency for open economic macro models. I demonstrate that the invoicing currency problem shares the same numerical property in a financial portfolio problem, which allows me to apply existing numerical method in the financial portfolio literature to calculate the zero-order invoicing share. The framework is flexible enough to be incorporated in a dynamic model with gradual price adjustment. I demonstrate the numerical procedure in solving currency invoicing in a general equilibrium example, and the result is consistent with the previous literature.

6 Appendix

6.1 First-order conditions for Rotemberg pricing

Assuming symmetry across firms producing different varieties. The first order condition for $P_{p,t}$ yields:

$$P_{p,t} = \frac{\theta MC_t(Y_{H,t} + \frac{S_{t-1}P_{p,t}}{e_t P_{f,t}} Y_{F,t})}{\Xi_{p,t}}, \tag{35}$$

where $\Xi_{p,t}$ is given by

$$\begin{aligned}
\Xi_{p,t} =& (\theta - 1)(1 - \frac{v_p}{2}(\pi_{p,t} - 1)^2)Y_{H,t} + (\theta - 1)S_{t-1}Y_{F,t} \\
&+ \frac{v_p}{2}(\pi_{p,t} - 1)^2 S_{t-1}(1 - \frac{\theta S_{t-1}P_{p,t}}{e_t P_{f,t}})Y_{F,t} \\
&- \theta\frac{v_l}{2}(\pi_{l,t} - 1)^2(1 - S_{t-1})\frac{S_{t-1}P_{l,t}}{P_{f,t}}Y_{F,t} \\
&+ v_p(\pi_{p,t} - 1)\pi_{p,t}(Y_{H,t} + S_{t-1}Y_{F,t}) \\
&- v_p E_t Q_{t,t+1}(\pi_{p,t+1} - 1)\pi^2_{p,t+1}[Y_{H,t+1} + S_t Y_{F,t+1}].
\end{aligned} \tag{36}$$

The first order condition for $P_{l,t}$ yields:

$$P_{l,t} = \frac{\theta MC_t \frac{(1-S_{t-1})P_{l,t}}{P_{f,t}} Y_{F,t}}{\Xi_{l,t}}, \tag{37}$$

where $\Xi_{l,t}$ is given by

$$\begin{aligned}
\Xi_{l,t} =& (\theta - 1)(1 - S_{t-1})e_t Y_{F,t} - \theta\frac{v_p}{2}(\pi_{p,t} - 1)^2 S_{t-1}(1 - S_{t-1})\frac{P_{p,t}}{P_{f,t}}Y_{F,t} \\
&+ v_l(\pi_{l,t} - 1)(1 - S_{t-1})e_t\pi_{l,t}Y_{F,t} \\
&+ \frac{v_l}{2}(\pi_{l,t} - 1)^2(1 - S_{t-1})e_t(1 - \frac{(1 - S_{t-1})\theta P_{l,t}}{P_{f,t}})Y_{F,t} \\
&- v_l E_t Q_{t,t+1}(\pi_{l,t+1} - 1)\pi^2_{l,t+1}(1 - S_t)Y_{F,t+1}.
\end{aligned} \tag{38}$$

The first order condition for S_t yields:

$$
\begin{aligned}
&E_t[Q_{t,t+1}P_{p,t+1}(1-\frac{\theta}{\theta-1}\frac{MC_{t+1}}{e_{t+1}P_{f,t+1}}\\
&+\frac{v_p}{2(\theta-1)}(\pi_{p,t+1}-1)^2-\frac{\theta}{\theta-1}\frac{v_p}{2}(\pi_{p,t+1}-1)^2S_t(\frac{P_{p,t+1}-e_{t+1}P_{l,t+1}}{e_{t+1}P_{f,t+1}}))Y_{F,t+1}]=\\
&E_t[Q_{t,t+1}e_{t+1}P_{l,t+1}(1-\frac{\theta}{\theta-1}\frac{MC_{t+1}}{e_{t+1}P_{f,t+1}}\\
&+\frac{v_l}{2(\theta-1)}(\pi_{l,t+1}-1)^2+\frac{\theta}{\theta-1}\frac{v_l}{2}(\pi_{l,t+1}-1)^2(1-S_t)(\frac{P_{p,t+1}-e_{t+1}P_{l,t+1}}{e_{t+1}P_{f,t+1}}))Y_{F,t+1}]
\end{aligned}
\tag{39}
$$

Equation 35, 37, 39, and together with equation 12 and the expressions for $\pi_{p,t}$ and $\pi_{l,t}$ contribute to a system of equations that solve the firm's profit maximization problem.

6.2 First-order approximation for Rotemberg pricing

The first order approximation for equation 35 and 37 leads to the standard Phillips curve, with a slight adjustment due the introduction of endogenous currency invoicing:

$$\hat{\pi}_{p,t}=\beta\hat{\pi}_{p,t+1}+\frac{\theta-1}{v_p}(r\hat{m}c_{p,t}+\frac{\bar{S}\bar{Y}_F}{\bar{Y}_H+\bar{S}\bar{Y}_F}\hat{\Delta}_{p,t}), \tag{40}$$

$$\hat{\pi}_{l,t}=\beta\hat{\pi}_{l,t+1}+\frac{\theta-1}{v_l}r\hat{m}c_{f,t}, \tag{41}$$

where $rmc_{p,t}=\frac{MC_t}{P_{p,t}}$ is the real marginal cost based on PCP, $rmc_{f,t}=\frac{MC_t}{e_tP_{f,t}}$ is the real marginal cost based on the foreign price, and $\Delta_{p,t}$ is the ratio between PCP and the foreign price $\frac{P_{p,t}}{e_tP_{f,t}}$. The Phillips curve for pure PCP(LCP) scenario can be obtained by setting $\bar{S}=1(\bar{S}=0)$

The first order approximation on equation 12 is similar to its counterpart in the static model:

$$\hat{\pi}_{f,t}=\bar{S}(\hat{\pi}_{p,t}-\hat{\epsilon}_t)+(1-\bar{S})\hat{\pi}_{l,t}, \tag{42}$$

where variable π_t represents the inflation rate for the respective price variable, and $\epsilon_t=\frac{e_t}{e_{t-1}}$ is the growth rate of nominal exchange rate.

The left-hand side and the right-hand side of the first order approximation to equation 39 cancel out with each other, which causes indeterminacy.

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